THE INFLUENCE OF DIFFERENTIATED MATHEMATICAL TASKS ON STUDENTS' LOGICAL-COMBINATORIAL THINKING IN ELEMENTARY MATHEMATICS TEACHING

Dunja Rakic – Bojan Lazic – Mia Maric

doi: 10.18355/PG.2021.10.1.7

Abstract

The aim of this research was to examine the influence of differentiated tasks on the development of logical-combinatorial thinking abilities of the first grade of primary school students, using the experimental-research method. The research was conducted on the sample (N=60) chosen from the population of first-grade students of primary school "Nikola Tesla" in Novi Sad. Obtained research results showed the high influence of differentiated mathematical problems on students' logical-combinatorial thinking and functional knowledge in elementary mathematics education. The results' practical implication mirrors the possibility of their application in the domain of the improvement of students' cognitive abilities and functional knowledge from the very beginning of the educational journey.

Key words

Logical-combinatorial thinking, differentiated problems, giftedness, elementary mathematics teaching, students

78

Introduction

In order for students' successfully adaptation to modern society and growing scientific and technological development, it is necessary to develop his or her creative abilities in approaching and solving problems. A high level of reasoning ability and developed logical-combinatorial thinking are the key characteristics of mathematical giftedness.

For the purpose of stimulation and adequate support for the development of mathematical thinking, as one of the primary goals of contemporary mathematics teaching, different forms of educational work that include additional engagement of this population of students, in the form of additional lessons, differentiation, and different further scientific research activities are applied. Additional lessons are flexible, and they include knowledge deepening in a specific domain, insight into the cause and effect relationships, solving problems, as well as finding new, creative, and original solutions (Lazić, 2011).

Goals and principles of mathematical education in a contemporary elementary school include acquisition of functional mathematical knowledge and skills, but also optimal development of cognitive abilities of each student. That urges the application of teaching models coordinated with individual differences between students, especially in elementary school education (Petrović, Pinter, 2002). That is possible to achieve through content differentiation according to the extent, depth, degree of difficulty, and abstractness. In that way, through the application of differentiated tasks

in the processing of mathematical content, the development of mathematical reasoning is maximally stimulated, respecting the individual capabilities and abilities of each student.

When identifying students with a high mathematical reasoning potential, one should begin with a set of inherited and acquired traits, as well as from determining the student's cognitive and conative-motivational traits (Lazić, 2011). Most researchers emphasize the developed logical-combinatorial thinking as an especially important component of mathematical giftedness and mathematical reasoning.

Researchers (Albrecht, Strydom, 2011, according to Rohaeti, Budiyanto, Sumarmo, 2014) define logical thinking as a conclusion of using consistent reasoning, cause, and effect thinking and generalization, evaluation, proving, analysis, and synthesis of certain activities. Logical-combinatorial thinking, as a component of studying mathematics, should be developed throughout the entire education, because it represents one of the goals of teaching mathematics. Besides this, the goal of teaching mathematics is developing the abilities which logically, systematically, critically, and correctly influence logical-combinatorial thinking (Rohaeti et al., 2014).

Studying complex scientific definitions, texts, classifications, functions, and combinatorial operations in mathematics demands special cognitive abilities. Mathematical reasoning specifically influences the development of logical-combinatorial thinking. Logical operations and set operations are, from a mathematical point of view, rooted in different psychological developmental processes (Nunes, Csapó, 2011). A great amount of attention is placed on students acquiring knowledge and being trained to perceive it critically, value it adequately, and apply it logically, as much as possible. In order for logical-combinatorial thinking to successfully develop, teachers and the whole knowledge acquisition process must explicitly incorporate abilities and skills of logical thinking (Maričić, Špijunović, Lazić, 2016). Children should develop interest and enthusiasm through education in primary school, as well as to learn how to adapt to situations, recognize and solve certain problems (Cvjetičanin, Anđić, 2017).

Combinatorial thinking is closely linked to different mathematical fields; however, suitable psychological processes are different for every field. Spontaneous stimulants coming from the environment cannot unite different mathematical fields and ideas; that is possible only through systematical mathematics teaching, which leads to the establishment of a higher level of connection between them. In this way, logical-combinatorial thinking is developed through solving certain mathematical problems in different fields (Rohaeti et al., 2014).

There are several types of problem situations that can be solved through combinatorial thinking. Problem situations include those situations which refer to fundamental counting problems or the situations in which diagrams, charts, and systemic lists are used. The application of fundamental counting strategies is used in solving tasks with a problem, which demands systematic questioning of alternative solutions. Other problems include tasks in which items from two or more separate sets are combined in all possible ways (English, 2005).

When it comes to logical operations, the Piaget's conclusions (1958) are relevant for mathematical education because, as he emphasized, the goal of mathematical education is to help students realize suggestions and interpret meanings by the structure of mathematical operations. Logical-combinatorial thinking is developed through teaching subjects, that is, through the analysis of the structure of logical operations and through emphasizing the relationship between structure and meaning. Combinatorial structures had a central role in Piaget's researches and logic development. Several later research projects also examined the influence of logical-combinatorial thinking development in mathematics and other school subjects (Kishta, 1979; Csapó, 1988, 2001, 2003; Schröder, Bödeker, Edelstein, Teo, 2000; Nagy, 2004). The care for early logical-combinatorial thinking development begins with the stimulation of mathematical education with well-structured exercises and tasks, which can be linked to other school subjects (Nunes & Csapó, 2011).

The results of research (Fischbein, Gazit, 1988, according to Batanero, Godino, Navarro-Pelayo, 1997) point to the effect of the development of combinatorial thinking with the use of specific combinatorial instruction, different games, and activities. It is discovered that ten-year-old students can learn some combinatorial ideas and, in that way, develop their logical-combinatorial thinking with the help of certain diagrams (Batanero, et al., 1997). Whether logical-combinatorial thinking can develop through solving mathematical problems was shown in research conducted at a university in Madison (USA). This research focused on a conceptual analysis of the influence of solving problems on logical-combinatorial thinking development. The data from this study indicate that certain models of task solving affect the effectiveness of mathematical education and logical-combinatorial thinking development (Lockood, 2012).

The theory about the development of geometric thinking (Van Hiele, 1986, according to Đokić and Zeljić, 2017), in which levels of thinking in students' work are stated, relies on students' logical reasoning and systematic thinking. It contains students' intuitive and inductive reasoning, which is based on patterns and regularities in solving problems set in new, unfamiliar situations (Đokić, Zeljić, 2017). This theory led the researchers to the idea of investigating the level of that type of thinking in students in lower grades of primary school. According to the results obtained from the tests and the scientific foundations of Van Hiele's theory, the researchers concluded that students develop their logical thinking through mathematics lessons and solving tasks (Vlasnović, Cindrić, 2014).

The process of solving mathematical problems creates an environment for the introduction of mathematical ideas, concepts, and procedures, which enhance the development of mathematical thinking (Jelić, Đokić, 2017). Also, knowing how to solve a mathematical problem represents the most significant characteristic of students' mathematical thinking, as well as their level of mathematical education (Dejić, Egerić, 2010).

The process of solving a mathematical problem for the purpose of logical-combinatorial thinking development should occur in four basic stages (Polya, 1966). For the successful exercising of mathematics, a good understanding of

a task, making a plan for problem-solving, plan realization, and problem-solving analysis are required. A particularly important role of tasks, reflected in the development of students' logical-combinatorial thinking, is in stimulating and implementing thinking operations, situation analysis, training for proper argumentation – proving claims and finding mistakes, creating mathematical models, and composing tasks independently.

In order to develop a student's task solving ability, a teacher must evoke a certain interest in that, provide the necessary time and develop the appropriate thinking operations (characteristic of particular stages of task solving). Additionally, students should master a general approach to solving tasks (Mladenović-Mirac, 2010).

A significant method of mathematical giftedness identification is through solving non-standard tasks. Students are assigned related non-standard tasks and, depending on their solution, different degrees of mathematical ability. Non-standard tasks can be classified in the following way: 1) tasks close to school teaching of mathematics, only more difficult – tasks from mathematics competitions; 2) interesting mathematical tasks – tasks which are very versatile when it comes to their content (Gusev, 2003, according to Dejić, Špijunović, Čebić, 2013).

One of the basic methods for recognizing students gifted for mathematics is testing, which is usually realized through different types of competition. Mathematics competition must be seen as a process that consists of three stages: preparation for the competition, the act of competition itself, and working with students after the competition (Špijunović, 1998).

All education systems in the world have their own developed network of competitions, but PISA-Programme for International Student Assessment and TIMSS-Trends in International Mathematics and Science Study is the most significant ones in international research. TIMSS relies on examination of formally acquired mathematical knowledge; that is, it represents teaching program, while PISA focuses on functional knowledge, that is, on the application of mathematics in a realistic context (Milinković, Lazić, 2018).

A competition which is very successfully organized in our country by The Mathematical Society "Archimedes" is called "Misliša". It has two levels: elementary level – for all applicants and national finals – for the most successful competitors at the elementary level, which is held in Belgrade. The goal of the competition "Misliša" is encouraging students to learn more about the content of school mathematics, developing students' logical-combinatorial thinking, developing students' interest and motivation to learn mathematics with pleasure, as well as spreading the mathematical culture among the young.

As elementary mathematics teaching offers different tasks in certain areas that can influence the development of logical-combinatorial thinking, the goal of this paper is to examine and process the influence of differentiated mathematical problems on first-grade students' logical-combinatorial thinking, manifested through the success at the competition testing.

Method

The problem of the research can be formulated as a question: How does elementary mathematics teaching influence the development of students' logical-combinatorial thinking? A contemporary child lives in a time where it is very important to think logically and critically perceive the world around it. The problem of this research is so significant, mostly because it is important that students encounter mathematical problems that influence their logical-combinatorial thinking from their earliest days in the educational system.

Related to this, the subject of the research is focused on differentiated mathematical problems, with the aim of determining their role and significance in developing logical-combinatorial thinking of first-grade students of elementary school, shown at the mathematical competition "Misliša". The degree of students' logical-combinatorial thinking development is operationalized through the success of solving suitable mathematical problems of logical-combinatorial type at this competition.

Since the founding of the competition in 2006 to 2018, the number of applicants rose to 470 000. The competition is held according to the regulations and operative instructions which every school receives, and any student from the first grade of elementary school to the fourth grade of secondary school can participate.

At the competition in 2019, 53 330 students from 637 schools (550 elementary schools) participated. For the first time since the competition "Misliša" was organized, still experimental, competition "Misliša" was held for the first graders too. Equal participation for the students of the first grade was enabled.

In this study, the descriptive and experimental-research method is applied. For the gathering of data and the interpretation of the obtained results, a descriptive research method was used. An experimental method is used in order to determine the differences between groups, that is, in order to examine the impact and effects of work on the experimental group, opposed to the control group of students. The experiment with the parallel group was used, in which the experimental group prepared for the competition mainly through differentiated tasks, while the control group of students prepared the usual way.

Gathering the data for this study was based on a research technique of testing - the students were tested through the mathematical competition "Misliša" after the preparation. The instrument used in the research was a standardized competition test, designed by the competent experts of The Mathematical Society "Archimedes". The test consisted of 15 tasks, differentiated according to the level of complexity in three groups of five tasks each. Tasks are scored with 3, 4, and 5 points, depending on which group, that is, in which level of complexity they are in. The majority of the tasks in the test require developed logical-combinatorial components: tasks with numbers and mathematical operations, logical tasks, tasks of combinatory nature, tasks with strings and drawings. The research instrument was designed so that the tasks measured the level of students' logical-combinatorial thinking development because every task had potential answers given, and a student

solved it using logical thinking by choosing only one answer, which he or she found correct.

Tests (tasks) in the competition are specific for each grade. At the elementary level, tests for the first and second grade of elementary school have 15 tasks each, differentiated according to three levels of complexity (three groups of five tasks each), scored with 3, 4, and 5 points (in a total of 60 points). Test for all of the remaining grades of elementary school has 25 tasks each, divided into three groups so that a student can score a maximum of 100 points. They are multiple-choice tasks so that a student chooses only one out of five answers (which considers correct) and marks it (writes it) on a separate answer sheet. Assignment time for first and second grade is limited to 75 minutes.

The study was conducted on a sample (N=60), chosen from the population of first-graders from Elementary School "Nikola Tesla" in Novi Sad, who participated in the competition "Misliša" (a deliberate sample). Since research is based on an experiment with parallel groups, we formed two equal groups of students – experimental (E) and control (C). The experimental group consisted of 30 students of two first grade classes, while the control group consisted of 30 students of the other two first grade classes from the same elementary school. It is a heterogeneous structure of groups of first-grade students, with approximately the same initial abilities, as determined by pedagogical-psychological service at the registration, so additional equalizing of the groups was performed according to the number and gender structure. The sample structure is presented in Table no. 1.

Table 1: Sample structure

Group	Boys	Girls	Total
Experimental	17	13	30
Control	16	14	30
Total:	33	27	60

In February and March, every school week, before the competition "Misliša", which was held on March 14, 2019, one additional mathematics lesson for the first-grade students was held. In total, six preparations for the competition were organized. The experimental group of students prepared for the competition mainly by working on differentiated tasks. Thereby, all students from the experimental group solved problems of two complexity levels. The examples of such tasks are found in the Appendix. The first task in the Appendix fits the second task level at the competition test, while the second level, according to the degree of complexity, could be classified as the second and third level of the competition task. In the additional classes, each student solves first-level tasks. After a certain amount of time, the teacher gives further explanation and checks whether the students completed the tasks correctly. In case they complete them correctly, the students go to the next level on their own. Otherwise, those tasks are explained frontally on the board. The accuracy of the second task is checked the same way. Besides the additional mathematics lessons, experimental group students prepared for the

competition by solving special tasks for homework on their own. Those tasks were from special collections of preparation tasks for mathematical competitions, published by the young mathematicians club Archimedes (2008), as well as "Mathematics for Younger Elementary School Classes: 100 Non-standard Tasks", also published by "Archimedes". In the collections, logical-combinatorial tasks were chosen first, and they were sorted according to their complexity. The students checked the accuracy of the tasks with their parents, as well as at their next additional lesson with the teacher. In that time, the control group prepared in an ordinary way, through the frontal solving of all types of problems only at their additional lessons. After the preparations for the competition in both sample groups of students, at the competition "Misliša", held in March 2019, testing of students was conducted simultaneously.

The objectivity of the test was ensured by placing each student in approximately the same test situation, by having independent examiners act on singular instructions, and by equally grading all tests based on one key. We have determined the logical and content validity of the tests by determining that the tests match the demands of first-grade *Mathematics* and the contents they refer to. Having analyzed the competition tests, we have determined that they are in accordance with learning outcomes expected from a first-grade student.

The data obtained from the research was processed with the use of statistical software package IBM Statistics SPSS20, using single-factor analysis of variance (ANOVA) for tracking the experimental program effects in the experimental group. All the tests were performed with a significance threshold of 0.05.

Results and discussion

In accordance with the set goal, efforts were made throughout the research to examine whether the students' preparation based on solving differentiated tasks achieves positive effects on students' logical-combinatorial thinking development and their abilities to solve mathematical problems on their own. After the preparations for the competition, both groups of first-grade students were tested at the mathematical competition "Misliša".

In the experimental group, the average number of points scored was 39.00 with the standard deviation of 9.001, while in the control group, the average number of points was by 15 less and it amounted to 24.133, while the standard deviation was statistically equal to the one in the experimental group and it was 8.935 ($F=0.985$, $p=0.968$). Applying the single-factor analysis (ANOVA), it was determined that the students from the experimental group had achieved a significantly better result ($F=41.210$, $p=0.000$). In the experimental group, 50% of the students scored 40 or more points, while that line in the control group was 26 points. The lowest number of points scored in the experimental group was 18, and the highest 56, while in the control group, the weakest achievement was from a student with no points at all, while the best one was from a student with 38 points. This means that more than half of the students from the experimental group scored better results than the best student in the control group.

Apart from the different approach in preparing students for the competition (experimental treatment), it was observed whether any other variable explains the difference in students' achievements. Using ANCOVA it was found that the treatment explains 57.1% of the total difference in the achievement, while gender explains none of the remaining parts. The review is in the Table no. 2.

Table 2: E and C group results according to gender structure

	Experimental		Control	
	average value	standard deviation	average value	standard deviation
boys	38.813	9.840	26.063	8.985
girls	39.214	8.304	21.929	8.669

The question is whether there is a difference in the achievement level of the two groups. As the tasks are scored with a different number of points depending on the level, they belong to, in this part of the analysis, the number of correctly completed tasks of the students at each level was observed. The results are presented in Table no. 3.

Table 3: Number of correctly solved tasks on different levels

	Experimental		Control		Significance
	Average value	standard deviation	average value	standard deviation	
elementary level	4.167	0.913	3.067	1.311	F=14.22, p=0.0004**
intermediate level	3.167	0.986	2.567	1.040	F=5.26, p=0.0255*
advanced level	2.767	1.305	0.933	0.868	F=41.05, p=0.0000**

*p<0.05; **p<0.01

All five tasks at the elementary level were correctly solved by 12 students from the experimental and 4 students from the control group. In the experimental group, most students (13) solved four tasks correctly, and in the control group, most students (9) solved three tasks correctly. The average number of correctly solved tasks in the experimental group is 4.167, while in the control group, the students solved one task less at average, that is, 3.067 average.

Most students (15) in the experimental group solved four tasks correctly, while in the control group, 16 students correctly solved three tasks each. In neither of the two groups did someone solve all five tasks. The students of the experimental group accomplished better results by solving 3.167 tasks, which is by half a task more than in the control group (2.567).

The greatest difference between groups can be seen in solving tasks at the advanced level. In the experimental group, most students (10) solved four tasks, while only two students solved none of the tasks from the advanced

level. In the control group, most students (14) solved one task, and even 10 students solved none of the tasks from the advanced level. The average number of completed tasks in the experimental group was 2.676, while in the control group, that number was by one and a half task less and amounted to only 0.933.

At all three levels, there is a statistical significance in the determined differences, which is presented in Chart no.3. As assumed, there is a difference inside every group in solving tasks at different levels. In both groups, the most accurately completed tasks are at the elementary level and the least at the advanced level, which can be seen in Graph no. 1.

Graph 1: Confidence intervals for the number of correctly completed tasks by levels.

In the experimental group, the number of correctly completed tasks at the elementary level is statistically significantly higher than the intermediate one ($t=4.007$, $p=0.001$) and at the advanced one ($t=4.815$, $p=0.000$), but there is no statistical significance in the number of tasks solved at the intermediate and advanced level ($t=1.340$, $p=0.185$). In the control group, there is no statistically significant difference in the number of tasks solved at the basic and intermediate level ($t=1.636$, $p=0.107$), but the number of tasks at the advanced level is statistically significantly smaller than the number of tasks solved at the intermediate ($t=6.603$, $p=0.000$) and basic level ($t=7.429$, $p=0.000$).

The results of this research have shown that students' preparation based on solving differentiated mathematical tasks can significantly influence students' logical-combinatorial thinking development in elementary mathematical education.

Observed in the context of previous theoretical assumptions about the development of logical-combinatorial thinking (Piaget, 1958; Polya, 1966), the findings of this research point to the significance of the preparation of students, which includes solving differentiated mathematical tasks for the development of their logical-combinatorial thinking and functional knowledge. Namely, the results obtained through this research confirm the previous theoretical assumptions about the significance of cognitive factors and suitable educational encouragement, which influence the development of

students' logical-combinatorial abilities (Maričić, Špijunović, Lazić, 2016; Nunes, Csapó, 2011; Petrović, Pinter, 2002; Rohaeti et al., 2014).

The results obtained from this study were expected to a large degree, in the light of previous research dealing with the effects of solving different types of tasks on the development of logical-combinatorial thinking (Kishta, 1979; Csapó, 1988, 2001, 2003; Schröder, Bödeker, Edelstein, Teo, 2000; Nagy, 2004). The authors examined different types of analytical and logical tasks in the domain of mathematics, using a specific combinatorial instruction, diagrams, different types of games and activities, and their influence on students' logical-combinatorial thinking abilities (English, 2005; Fischbein & Gazit, 1988, according to Batanero et al., 1997). The results of this research complete past knowledge and previous research findings by confirming the significance of solving differentiated mathematical tasks on students' cognitive development in the domain of logical-combinatorial thinking.

Therefore, the results of this research continue the results of previous research studies on the meaning and effects of specific types of tasks, instructions, and stimulating learning context on the cognitive development and intellectual abilities of early elementary school-age children. Previous studies have also shown that certain methods of solving mathematical tasks stimulate the development of students' logical-combinatorial thinking (Lockwood, 2012; Vlasović & Cindrić, 2014). However, the specific contribution of this research mirrors in the fact that it showed the significance of the competition preparation process (through the application of specific mathematical tasks), the role of the competition itself in students' cognitive development, and especially the effects of applying specific differentiated mathematical tasks in the development of logical-combinatorial thinking of younger elementary school students. Significantly better results achieved by the experimental group, who had solved differentiated mathematical tasks, point to a unique contribution of differentiated mathematical tasks to the development of students' logical-combinatorial thinking. Additionally, the examination context itself shows the role of the competition and student preparation in the development of specific cognitive abilities at a younger school age, which is of great significance for the entire cognitive development and intellectual potential of students in the future development phases.

Conclusion

The development of functional knowledge and logical-combinatorial thinking are the main goals of elementary mathematical education. Bearing this in mind, it is necessary to apply such teaching methods in elementary education, which will, with respect to students' individual differences, enable the optimal development of cognitive abilities in each of them. The Standards that have been achieved with descriptions at three levels (basic, intermediate, and advanced) are attributed to "to assist teachers in preparing teachers to use individual student characteristics" (Dejić & Milenković, 2016: 1).

Since elementary mathematics teaching offers different tasks in specific areas, which can influence the development of logical-combinatorial thinking, the aim of this research was the examination and analysis of

differentiated mathematical tasks' influence on the development of logical-combinatorial abilities of first-grade elementary school students. The results acquired in the experimental research show that differentiated tasks significantly and positively influence the development of logical-combinatorial thinking of students in elementary mathematics teaching.

If we take into consideration that these tasks are present at international competitions (such as PISA, TIMSS) to a considerable extent, the acquired results support the application of these mathematical tasks in elementary mathematics teaching even more. Besides, the degree of success in solving these tasks obviously points to the level of mathematical literacy. It is justified to assume that an optimal level of student activity is achieved through solving differentiated mathematical tasks, which secures an adequate atmosphere for the development of creative thinking and logical reasoning in the lower grades of elementary school. Logical-combinatorial thinking developed this way is a significant component of cognitive abilities which a student can apply in different everyday life situations, which require practical actions.

A potential limitation of this research is the fact that the tasks used at the competition "Misliša" are of a closed-ended type (the answers are given by choosing one of the provided options). Therefore, with the aim of overcoming this limitation, it would be useful to include the data acquired by solving open-ended tasks as well (such as PISA testing), by which the possibility of guessing the solution or solving the task through the elimination process would be reduced.

Speaking of potential limitations of this study, from the methodological point of view and the aspect of the application of acquired results, in future research, it would be preferable to examine the individual influence of preparation classes (including differentiated mathematical tasks), effects of homework, as well as their potential combined influence and the interaction of these factors, on the development of students' logical combinatorial thinking and success at the competition.

According to the results acquired this way through the research, new ideas appear for the development of students' logical-combinatorial thinking. Moreover, new possibilities appear for the examination of other stimulating factors, which can be related to the development of students' logical-combinatorial thinking. In future research, it would be significant to examine how other types of tasks, at another age, influence logical-combinatorial thinking development. New ideas for research in this domain can also refer to the question: how to do differentially designed mathematical tasks, through different forms of work, influence students' cognitive abilities.

Bibliographic references

BATANERO, C. – GODINO, J. D. – NAVARRO-PELAYO, V. 1997. Combinatorial Reasoning and its Assessment. In: Gal, I. & Garfield, J. B. (Eds.), *The Assessment Challenge in Statistics Education* Granada: International Statistical Institute. pp. 239-252.

- CVJETICANIN, S. – ANĐIĆ, B. 2017. Teacher's opinion about the implementation of inclusive education in teaching integrated natural sciences. *Teme*, vol. 41, n. 4, pp. 999–1016. doi: 10.22190/TEME1704999C
- DEJIC, M. – SPIJUNOVIC, K. – CEBIC, S. 2013. Nestandardni zadaci u funkciji identifikovanja matematičkih sposobnosti [Nonstandard Tasks in Terms of Identifying Mathematical Abilities]. *Collection – Preschool Teacher Training College*, vol. 18, pp. 96–111.
- DEJIC, M. – EGERIC, M. 2010. Metodika nastave matematike [Mathematics Teaching Methodology]. Belgrade: Teacher Education Faculty.
- DEJIC, M. – MILENKOVIC, V. 2016. Standardi postignuća učenika u funkciji efikasne diferencirane nastave matematike [Standards of Students' Achievements Functioning in Efficient Differentiated Teaching Mathematic]. *Inovacije u nastavi*, vol. 29, n. 2, 15–24. doi: 10.5937/inovacije1602015D
- ĐOKIĆ, O. – ZELJIC, M. 2017. Teorije razvoja geometrijskog mišljenja prema Van Hilu, Fišbajnu i Udemon-Kuzniaku [Theories of Geometric Thinking Development by Van Hiele, Fishbein and Udemon-Kuźniak]. *Teme*, vol. 41, n. 3, pp. 623–637. doi: 10.22190/TEME1703623D
- ENGLISH, L. 2005. Combinatorics And The Development Of Children's Combinatorial Reasoning. In: Jones, G. A. (Eds.), *Exploring Probability in School: Challenges for Teaching and Learning*, New York: Springer. pp. 121–141. doi: 10.1007/0-387-24530-8_6
- JELIĆ, M. – ĐOKIĆ, O. 2017. Ka koherentnoj strukturi udžbenika matematike – analiza udžbenika prema strukturnim blokovima TIMSS istraživanja [Towards coherent structure of Mathematics textbooks – analysis of textbooks according to structural blocks of TIMSS research]. *Inovacije u nastavi*, vol. 30, n. 1, pp. 67–81. doi: 10.5937/inovacije1701067J
- LAZIC, B. 2011. Oblici obrazovne podrške darovitih učenika [The forms of Educational Support for Gifted Students]. *Inovacije u nastavi*, vol. 24, pp. 106–117.
- LOCKWOOD, E. 2012. A model of Students' Combinatorial Thinking: The Role of Sets of Outcomes. In: Brown, S., Larsen, S., Marromgelle, K., Oehrtman, M. (Eds.), *15th Annual Conference on Research in Undergraduate Mathematics Education*, Portland, Oregon. pp. 1–15
- MARICIC, S. – SPIJUNOVIC, K. – LAZIC, B. 2016. The Influence of Content on the Development of Students' Critical Thinking in the Initial Teaching of Mathematics. *Croatian Journal of Education*, vol. 18, n. 1, pp. 11–40. doi: 10.15516/cje.v18i1.1325
- MILINKOVIC, B. – LAZIC, M. 2018. Postignuća učenika na TIMSS i PISA ispitivanju kao smernica za izmene u nastavnom programu matematike [Students' Achievements at the TIMSS and PISA Examination as a Guide for Changes in the Mathematics Curriculum]. *Inovacije u nastavi*, vol. 31, n. 3, pp. 74–87. doi: 10.5937/inovacije1803074M
- MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGICAL DEVELOPMENT 2015. Stručno uputstvo o organizovanju takmičenja i smotri učenika osnovne i srednje škole [Expert guidance on organizing competition and monitoring high school and elementary school students].

Retrieved May 18 2019. Available online: <http://www.mpn.gov.rs/wp-content/uploads/2015/08/Stru%C4%8Dno-uputstvo.docx>

NUNES, T. – CSAPO, B. 2011. Developing and Assessing Mathematical Reasoning. In: Csapó, B. & Szendrei, M. (Eds.), Framework for Diagnostic Assessment of Mathematics, Budapest: Nemzeti Tankönyvkiadó. pp. 17-56.

PETROVIC, N. – PINTER, J. 2002. Opšta metodika nastave matematike [General Teaching Methodology]. Sombor: Faculty of Education.

POLYA, G. 1966. Kako ću riješiti matematički zadatak [How do I solve a Math Problem]. Zagreb: Školska knjiga.

ROHAETI, E. – BUDIYANTO, A. – SUMARMO, U. 2014. Enhancing Students' Mathematical Logical Thinking Ability and Self-regulated Learning Through Problem-based Learning. International Journal of Education, vol. 8, n. 1, pp. 54-63.

SPIJUNOVIC, K. 1998. Takmičenje kao metod identifikacije i podsticanja učenika potencijalno darovitih za matematiku [Competition as a Method Identification and Stimulation of Students Potentially Gifted for Mathematics]. Collection – Preschool Teacher Training College, vol. 4, pp. 263-273.

VLASNOVIC, H. – CINDRIC, M. 2014. Razumijevanje geometrijskih pojmova i razvitak geometrijskog mišljenja učenika nižih razreda osnovne škole prema Van Hieleovoj teoriji [Understanding Geometric Terms and the Development of Lower Graders' Geometric Reasoning According to Van Hiele's Theory]. Školski vjesnik: Journal for Pedagogic Theory and Practice, vol. 6, n. 1-2, pp. 37-51.

Dunja Rakić
Primary school “Nikola Tesla”
Futoski put, 25a, Novi Sad 21000
Serbia
dunja.rakic@yahoo.com

assoc. prof. dr. Bojan Lazić
Department Mathematics and methodic of mathematic teaching
University of Novi Sad, Faculty of Education in Sombor
Podgoricka 1, Sombor 25 000
Serbia
bojan.lazic@pef.uns.ac.rs

prof. dr. Mia Marić
Department Social sciences
University of Novi Sad, Faculty of Education in Sombor
Podgoricka 1, Sombor 25 000
Serbia
mia.maric@pef.uns.ac.rs

APPENDIX
Review of differentiated tasks

TASK N.	FIRST LEVEL	SECOND LEVEL
1.	Jana has made a bouquet of 4 daffodils, 3 tulips, 5 roses and 3 carnations. How many flowers are there in Jana's bouquet? a) 11 b) 12 c) 13 d) 14 e) 15	Two pigs and three ducks, how many beaks and how many legs are there. a) 3 beaks, 10 legs b) 2 beaks, 12 legs c) 3 beaks, 14 legs d) 3 beaks, 16 legs e) 5 beaks, 14 legs
2.	Each one of Luka's 8 trucks is missing one wheel. How many wheels are there on all Luka's trucks? a) 33 b) 28 c) 24 d) 21 e) 16	Bojan has 10 cars. Miša brought him 4 more, which were missing two wheels each. How many wheels are there on all cars in total? a) 46 b) 38 c) 48 d) 54 e) 24
3.	Grandma Zora's yellow hen laid 8 chickens, while her white hen laid 4 chickens more. How many chickens does Grandma Zora have in total? a) 8 b) 12 c) 20 d) 24 e) 28	Grandma Maša has a hen which laid twice as many eggs as Grandma Vera's hen. Grandma Vera's hen laid 7 eggs. How many eggs do Grandma Maša and Grandma Vera have in total? a) 14 b) 9 c) 20 d) 21 e) 24
4.	On the right river bank there are 3 boys and a boat which can take two boys at a time. How many times do the boys have to cross the river in order for all three of them to get to the other bank? a) 1 b) 2 c) 3 d) 4 e) 5	When they were leaving, 3 boys shook hands. Each one of them shook all of the other's hands. How many handshakes were there if each one of them shook all of the other's hands? a) 1 b) 2 c) 3 d) 4 e) 5
5.	Brother's and sister's combined age is 7. What will their combined age be in 3 years? a) 13 b) 7 c) 10 d) 14 e) 12	Mom's, dad's, brother's and sister's combined age is 60. What will their combined age be in 3 years? a) 63 b) 72 c) 77 d) 66 e) 74
6.	How many wings are there in four pairs of pigeons? a) 8 b) 10 c) 12 d) 14 e) 16	Zoran had 15 pigeons. He gave 4 pairs of pigeons to his friend. How many does he have left? a) 7 b) 19 c) 9 d) 16 e) 11

7.	<p>How many squares and how many rectangles are there in the picture?</p> <p>a) s: 5 r: 3 b) s: 3 r: 3 c) s: 2 r: 3 d) s: 1 r: 1 e) s: 3 r: 1</p>	<p>How many rectangles are there in the picture?</p> <p>a) 8 b) 10 c) 9 d) 11 e) 7</p>
8	<p>What is the next member of the string: 19, 1, 17, 1, 15, 1, 13, __</p> <p>a) 1 b) 11 c) 2 d) 12 e) 14</p>	<p>What is the next member of the string: 2, 5, 4, 7, 6, 9, 8, __</p> <p>a) 10 b) 11 c) 12 d) 13 e) 14</p>
9.	<p>Gara runs faster than Floki, and Floki is faster than Džeki. Which dog is the fastest?</p> <p>a) Gara b) Džeki c) they are all equally fast d) it cannot be determined</p>	<p>Bora is shorter than Luka, and Luka is taller than Stefan. Who is the tallest? Can it be determined who is the shortest?</p> <p>a) Bora is the tallest, Luka is the shortest b) Bora is the tallest, Stefan is the shortest c) Luka is the tallest, Bora is the shortest d) they are all the same height e) Luka is the tallest, it is impossible to determine who is the shortest</p>
10.	<p>At Marko's birthday cake 8 candles were burning. One candle burns for 8 minutes. In how many minutes will all the candles burn out, providing they were all lit at the same time?</p> <p>a) 6 min. b) 3 min. c) 12 min. d) half an hour e) 8 min.</p>	<p>Six candles were burning. Four got extinguished by the wind. How many candles are there left?</p> <p>a) 6 b) 4 c) 2 d) 10 e) 0</p>
11.	<p>A book is missing sheets with pages marked 5-18. How many sheets are missing?</p> <p>a) 6 b) 7 c) 8 d) 9 e) 10</p>	<p>Milica is missing one page in a book. Her sister gave her an interesting task to figure out the number of a missing page. She told her to think of the largest single-digit number, and then increase it by 10, then to reduce the sum by the smallest even number. Which page number was missing?</p> <p>a) 15 b) 7 c) 12 d) 9 e) 17</p>
12.	<p>Acrobat digit-if you place it on its head, it reduces by 3! Which digit is that?</p> <p>a) 0 b) 8 c) 3 d) 6 e) 9</p>	<p>In which digit does the sum of all single-digit numbers smaller than 6 end?</p> <p>a) 0 b) 5 c) 1 d) 7 e) 8</p>